PARTIAL DIFFERENTIAL EQUATIONS: ANALYSIS, NUMERICS AND APPLICATIONS TO FLOODS AND TSUNAMIS

A CIMPA – ICIAM – IMU-CDC Research School

Institute of Mathematics
University of the Philippines Diliman
23 June – 04 July 2014

Partial Differential Equations: Analysis, Numerics and Applications to Floods and Tsunamis

A CIMPA – ICIAM – IMU-CDC RESEARCH SCHOOL

The Research School was held on 23 June – 04 July 2014 at the Institute of Mathematics, University of the Philippines Diliman in Quezon City, Philippines. The idea of conducting a school on partial differential equations was conceived during one of Patrizia Donato's visits to UP Diliman as part of her on-going research collaboration with Marian Roque.

The school was intended for graduate students who wish to specialize in partial differential equations. The lectures were conceptualized to arm students with the necessary tools for the analysis of PDEs, their numerical approximation and applications in the modeling natural phenomena. In particular, models of waves and tsunamis were discussed as these would be meaningful in the Southeast Asian region.

THE ORGANIZERS

The scientific and administrative coordinators were Patrizia Donato of the University of Rouen in France and Jose Ernie Lope of UP Diliman. Brigitte Lucquin and Doina Cioranescu (University of Paris VI) also contributed significantly in the conceptualization of the lectures and in selecting the non-Filipino participants to the Research School.

Doina Cioranescu is an old friend of the Institute and was among the organizers of a CIMPA School held at UP in the late 1990s. Patrizia Donato and Brigitte Lucquin first visited UP more than five years ago through the EU-funded program IMAMIS; they both gave a series of lectures on PDEs over a span of two weeks. Even after the IMAMIS Program, Marian Roque continued the collaboration with Patrizia and Doina and the three of them co-authored a book on PDEs.

On the local end, the school would not be possible without the efforts of Marian Roque, who is also the Director of UP Diliman's Institute of Mathematics. She was assisted by the

following faculty members: Jasmin Mae Santos, May Anne Tirado, Kelvin Lagota, Bituin Cabarrubias, Dennis Leyson, and many others.

THE LECTURERS

Seven renowned professors, five (5) males and two (2) females, were invited to give lectures at the school. Four (4) came from France, and one (1) each from the Netherlands, the U.S.A. and Japan. Listed below, in alphabetical order, are their names, their affiliations and the titles of their talks:

1. Adel Blouza (University of Rouen, France; Variational problems)
2. Patrizia Donato (University of Rouen, France; Sobolev spaces)
3. David Lannes (Ecole Normale Supérieure de Paris, France; The water waves equations)
4. Brigitte Lucquin (University Pierre et Marie Curie, France; Numerical approximation of some partial differential equations)
5. Masahisa Tabata (Waseda University, Japan; Numerical analysis of flow problems)
6. Bogdan Vernescu (Worcester Polytechnic Institute, U.S.A.; Introduction to the modeling of viscous fluids)
7. Marcel Zijlema (Delft University of Technology, The Netherlands; Numerical modeling of waves and applications to wind waves, hurricanes and flooding waves)

The abstracts of their talks as well as the schedule of the lectures may be found at the appendix. As can be noticed in the titles of the talks, some of the speakers gave elementary lectures on PDEs for the benefit of those who are new to the field. Towards the latter half of the school, the talks shifted to more advanced topics that can be the seeds of future research investigations.

THE PARTICIPANTS

A total of 55 students were supposed to take part in the Research School but four (4) eventually did not make it for various reasons. Of the 51 participants, 37 were Filipinos and 14 were from the following countries: Cambodia (5), India (1), Indonesia (3), Japan (1), Nepal (2), Nigeria (1) and Pakistan (1). It must be noted, however, that three (3)

of the Cambodians are currently enrolled at the Institute of Mathematics and one (1) Filipino is a graduate student in Japan.

All the participants from abroad, except for the lone Japanese, were fully supported by the Research School. The Japanese student paid for his own airfare and accommodations but was not charged for lunch and snacks for the whole duration of the school.

Of the Filipino participants, 25 were from Metro Manila and 12 were not; the participation of these 12 was supported in full (transportation, board and lodging).

There was a relatively good gender balance among the participants: there were a total of 16 females and 35 males.

The complete list of participants may be found at the appendix.

THE VENUE

The lectures were held at a medium-sized room on the 3rd floor of the Math Building Annex. The room has a wall-length blackboard, an LCD projector and a projection screen. There was also the possibility of using a microphone if necessary.

Coffee and snacks were served just outside this room. During the breaks, participants usually ate and chatted at the hallway until it was time for the next lecture. Lunch was served on the same floor, at an open-air portion of the building. The air-conditioned room adjacent to the lecture room was available during rainy days and also for those who didn't wish to dine al fresco.

Wi-Fi was made available for everyone. Four wireless routers at the Math Building Annex were configured for the sole use of the school participants. Most, if not all, of the participants brought their own laptops or tablets so there was no need to set up a computer for email or other needs.

BOARD AND LODGING

The speakers and the non-Filipino participants (except for one who arrived after the start of the school) were met at the airport by members of the local organizing committee and brought to their respective places of stay.

The speakers were billeted either at the University Hotel or at Balay Kalinaw. There is a restaurant at the University Hotel so those staying at Balay (just a couple hundred meters away from the Hotel) could just walk over to the Hotel to eat breakfast or dinner.

The foreign participants stayed at the UP NISMED Hostel, which was about five minutes away from the Institute on foot. On the other hand, the non-Metro Manila participants stayed at the SOLAIR Hostel, also inside the UP Diliman Campus.

Participants were asked beforehand of their food preferences. Except at the initial day of the school, halal and vegetarian meals were available. Those without restrictions had a fair sample of Filipino viands and snacks during their two-week stay.

The organizers did not make arrangements for breakfast and dinner nor for weekend meals. Instead, the participants and speakers were given a meal allowance. Cheap meals could be taken at the canteen of NISMED Hostel or at the Shopping Center of UP Diliman. Those wanting more options could go to the nearby UP Town Center or to one of the middle-range restaurants on campus.

OTHER ACTIVITIES

A short opening program was held at an auditorium of the Institute of Mathematics before the start of the school. The Dean of the College of Science, Professor Jose Maria Balmaceda (also a mathematician), opened the program. Prof Alain Damlamian gave a message as CIMPA Representative while Patrizia Donato and Jose Ernie Lope spoke as Scientific and Administrative Coordinators.

The School's welcome dinner was held at the Executive House (official residence of the President of the UP System). The Chancellor of UP Diliman, Professor Michael Tan, accepted the organizers' invitation to give a message to the lecturers and participants.

The two Wednesday afternoons were purposely left free for short trips within Metro Manila. On the first, the participants and some speakers went to Intramuros, the seat of power during the long Spanish occupation of the Philippines. On the second, some participants went to a nearby mall to buy souvenirs or other personal items. A day-long excursion was organized on Saturday, June 28. The participants went to Tagaytay (about 50 km away, two hours by car) to see Taal Volcano, an active volcano inside a scenic lake.

One of the speakers took a day-trip to Corregidor Island, the last stronghold of the Filipinos and Americans during the Second World War.

SUPPORT FOR THE SCHOOL

The School received tremendous support from CIMPA in the amount of 7,570 euros. In addition, the International Mathematical Union – Committee for Developing Countries (IMU-CDC) gave 2,000 euros while the International Council for Industrial and Applied Mathematics (ICIAM) gave 3,500 US dollars. On the local side, the Institute of Mathematics and UP's Office for Institutional Linkages each gave 100,000 pesos (about 2,300 US dollars) while the Philippine Council for Industry, Energy and Emerging Technology Research and Development (PCIEERD) gave 50,000 pesos (about 1,150 US dollars).

Other than these monetary contributions, the school organizers would like to acknowledge the free use of the Institute of Mathematics' various facilities and resources. Thanks also go to the junior faculty members and institute staff who volunteered time and effort in meeting the guests at the airport, showing them around, organizing the excursions and helping in the other activities of the school.

Prepared by:

Jose Ernie C. Lope and **Patrizia Donato**
Administrative and Scientific Coordinators

and

Marian P. Roque
Finance Officer

VARIOUS PHOTOS

Prof. David Lannes giving a lecture

The typical lecture hall scene

During the Welcome Dinner

Local organizers with UPD Chancellor and CS Dean

Excursion to Taal Volcano

Time to say "Good-bye!"

APPENDIX

I. Abstracts of Talks

The following are the abstracts of all the lectures given at the Research School.

A. Variational problems (Adel Blouza)

The course deals with variational PDEs, where weak solutions are considered, verifying a weak formulation of the equations and belonging to suitable spaces of functions (Sobolev spaces).

In a first part we recall some basic properties on bilinear form on Hilbert spaces, and we prove the Lax-Milgram Theorem. Then, we apply it to prove existence, uniqueness and a priori estimates for weak solutions of second order linear elliptic equations in the divergence form. We consider three types of boundary conditions: Dirichlet, Neumann, and Robin.

In a second part we treat the corresponding evolution problems: the heat equation and the wave equation. To do that, we define and recall some results on vector-valued Sobolev spaces.

B. Sobolev Spaces (Patrizia Donato)

The course is devoted to a presentation of some classes of Sobolev Spaces which is the main tool in the study of variational PDEs. It starts with some examples as motivation.

In a first part we briefly recall some simple notion about distributions, then define the Sobolev space $H^1(W)$. We give its main properties, in particular the completeness, a density result, the notion of trace on the boundary of a domain. We also characterize this space, for $= R^n$, using the Fourier transform. Then, we define the space $H_0^1(W)$ and prove its main properties, in particular the Poincaré inequality and some related trace results.

In the second part, we state and prove some Sobolev embedding theorems and some compactness results.

C. The water waves equations (David Lannes)

This course will be devoted to a theoretical approach of the description of waves. We will first discuss various formulations of the equations governing the problem (free surface

water waves equations) and after giving a brief description of their structure, we will explain how to construct solutions to these equations.

The second part of the course will be devoted to an asymptotic description of these solutions through simpler asymptotic models. We will introduce several concepts allowing one to handle the asymptotic descriptions of water waves in the so called shallow water regime, and show how one can derive most of the models used in oceanography to model the transformation of waves in coastal regions.

Numerical aspects will also be addressed through a practical. We will particularly insist on the importance of the modelling of dispersive effects for strong topographic variations.

D. Numerical approximation of some Partial Differential Equations (Brigitte Lucquin)

We present different numerical approximations of Partial Differential Equations. The course is divided into two parts.

In the first part, we present the finite element method which is well adapted to the numerical approximation of elliptic boundary value problems, once these problems have been transformed into their variational form. We first recall the continuous problem and its variational formulation, then we introduce the discrete problem in a general frame. We analyze more precisely the method in the one-dimensional case (explicit construction of the discrete variational space, computation of the approximated solution and convergence analysis). We also describe the method in the two-dimensional case, using either rectangles or triangles. Last, we generalize to systems: the Lame system and the Stokes system.

The second part is devoted to the numerical approximation of some evolution problems of parabolic or hyperbolic type. We first study the case of the heat problem and describe a finite difference approximation of it but also a mixed finite differences-finite elements approximation. Last, we describe and analyze the finite difference approximation of the wave equation.

E. Numerical Analysis of Flow Problems (Masahisa Tabata)

In this lecture numerical analysis of flow problems are discussed. More precisely, the theory and practice of the finite element method are given. We deal with Poisson problem, convection-diffusion problem, Stokes problem, Navier-Stokes problem, and two-fluid flow problems.

At first we show some bilinear forms, boundary conditions and weak formulations. We introduce finite element spaces and apply Galerkin approximations to get Galerkin finite element methods. In the cases of high-Peclet or high-Reynolds numbers Galerkin finite element methods cause numerical oscillations, which lead us to upwind schemes or Galerkin-characteristics schemes. Some recent results on Galerkin-characteristics schemes are presented.

Next we introduce the saddle-point formulation to solve Stokes problems. Navier-Stokes problems are solved by combining the Stokes solver and Galerkin-characteristics schemes. Finally as an application we consider two-fluid flow problems governed by the Navier-Stokes equations with different densities and viscosities.

F. Introduction to the Modeling of Viscous Fluids (Bogdan Vernescu)

Modeling of viscous fluids is fundamental to the understanding of a wide range of problems in engineering, geophysics and atmospheric sciences. Starting from the fundamental principles of continuum mechanics, the lectures will introduce the Navier-Stokes equations for incompressible, viscous fluids and will discuss the solutions for some elementary examples.

The validity for the Stokes equations will be justified as a result of a dimensional analysis.

The formulation of the Navier-Stokes equations in Sobolev spaces will be introduced and the existence and uniqueness results will be discussed, for both stationary Stokes and Navier-Stokes equations. Results for the time dependent case will be presented and a brief discussion of unresolved problems related to the turbulent flow will end the course.

G. Numerical modelling of waves and applications to wind waves, hurricanes and flooding waves (Marcel Zijlema)

In this course we shall deal with two types of numerical wave modelling.

The first one is based on the shallow water equations with vertical acceleration. These equations are derived from the Navier-Stokes equations with the free surface as a part of the solution. This type of modelling enables to simulate individual waves in both time and domain. As such, all kind of physical processes can be described on virtually all time- and space scales, e.g. shoaling, refraction, diffraction, wave-wave interactions, wave-current interactions, etc. Some numerical aspects will be discussed, and some practical issues concerning the modelling of waves will be outlined as well. For this type of wave modelling a practical will be done using the model SWASH of Delft University of Technology

(swash.sf.net). Some examples of wave transformation, wave flooding and tsunamis will be presented.

Another type of numerical wave modelling is based on the spectral energy balance and the waves are described by means of a spectrum. As such, only wave amplitudes, frequencies and directions can be computed whereas wave phases are supposed to be random. This type of wave modelling is very efficient in the sense that the distribution of wave energy in a relative large basin of order of 1000 km by 1000 km can be simulated. Many physical processes like wave breaking, bottom friction, white capping, wind input, and wave-wave interactions are parameterized, while refraction and frequency-shifting due to bottom and current variations are derived from the linear wave theory. A well-known example is the Delft's SWAN model (swanmodel.sf.net). You will be acquainted with this model by means of a practical.

Some examples of hurricanes and super storms will be presented as well.

II. The School Program

The lectures at the school were conducted according to the following schedule:

Week 1

Time	Monday	Tuesday	Wednesday	Thursday	Friday
8:00-9:00	Opening				
9:00-10:30	Donato	Donato	Donato	Lucquin	Vernescu
10:30-11:00	BREAK	BREAK	BREAK	BREAK	BREAK
11:00-12:30	Blouza	Blouza	Blouza	Vernescu	Lucquin
12:30-14:00	LUNCH	LUNCH	LUNCH	LUNCH	LUNCH
14:00-15:30	Donato	Donato		Lucquin	Lucquin
15:30-16:00	BREAK	BREAK	EXCURSION	BREAK	BREAK
16:00-17:30	Blouza	Vernescu		Vernescu	Tabata

Week 2

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00–10:30	Tabata	Tabata	Lannes	Zijlema	Lannes
10:30–11:00	BREAK	BREAK	BREAK	BREAK	BREAK
11:00–12:30	Lannes	Lannes	Zijlema	Lannes	Zijlema
12:30–14:00	LUNCH	LUNCH	LUNCH	LUNCH	LUNCH
14:00–15:30	Tabata	Zijlema		Zijlema	
15:30–16:00	BREAK	BREAK	EXCURSION	BREAK	BREAK
16:00–17:30	Zijlema	Zijlema		Lannes	

III. List of participants

A. Coming from abroad

	Name	Sex	University	Country
1	Acharaya, Saraswati	F	Kathmandu University	Nepal
2	Ahmad, Sohail	M	Abdus Salam School of Mathematical Sciences	Pakistan
3	Erwina, Novry	F	Institut Teknologi Bandung	Indonesia
4	Wiraningsih, Eti Dwi	F	Jakarta State University	Indonesia
5	Husain, Akhlaq	M	The LNM Institute of Information Technology	India
6	K.C., Gokul	M	Kathmandu University	Nepal
7	Lin, Mongkolsery	M	Mahidol University	Cambodia
8	Mohammad, Rhudaina	F	Kanazawa University	Philippines
9	Mungkasi, Sudi	M	Sanata Dharma University	Indonesia
10	Nwaigwe, Chinedu	M	Rivers State University of Science and Technology	Nigeria
11	Tann, Chantara	F	Institute of Technology Cambodia	Cambodia
12	Uchiumi, Shinya	M	Waseda University	Japan
13	Narith, Seak	M	UP Diliman	Cambodia
14	Say, OL	M	UP Diliman	Cambodia
15	Sotheara, Veng	M	UP Diliman	Cambodia

The last three participants are Cambodians who are currently studying at the Institute of Mathematics of UP Diliman. Note that Rhudaina Mohammad is a Filipina but is currently a PhD student in Japan. Note also that Shinya Uchiumi is not among those funded by the school; he is a Japanese PhD student who came on his own (upon the recommendation of Prof Tabata).

B. Coming from outside Metro Manila

	Name	Sex	University
1	Addawe, Joel	M	UP Baguio
2	Artes Jr., Rosalio	M	Mindanao State University-Iligan Institute of Technology
3	Bacani, Jerico	M	UP Baguio
4	Caga-anan, Randy	M	Mindanao State University-Iligan Institute of Technology
5	Catinan, Filame Joy	F	UP Visayas-Miag-ao
6	Cortez, Mark Jayson	M	UP Los Baños
7	Faina,Lindley Kent	M	UP Visayas-Miag-ao
8	Gentuya, Junmar	M	UP Visayas-Tacloban College
9	Jose, Editha	F	UP Los Baños
10	Libo-on, Jeoffrey	M	UP Visayas-Miag-ao
11	Mamplata, Jonathan	M	UP Los Baños
12	Peralta, Gilbert	M	UP Baguio

C. Coming from Metro Manila (National Capital Region)

	Name	Sex	University
1	Alas, Roseanne	F	UP Diliman
2	Almocera, Alexis	M	UP Diliman
3	Antonio, Victor Andrew	M	Ateneo de Manila University
4	Aviles II, Mathithias	M	UP Diliman
5	Babierra, Ariel	M	UP Diliman
6	Bargo, Cristina	F	UP Diliman
7	Borja, Genesis John	M	UP Diliman
8	Boydon, Kai	F	UP Diliman
9	Cabarubias, Bituin	F	UP Diliman
10	Cheng, Hanz Martin	M	Ateneo de Manila University
11	Dela Cruz, Laarni	F	UP Diliman
12	Fulgencio, Rheadel	F	UP Diliman
13	Lagota, Kelvin	M	UP Diliman
14	Leyson, Dennis	M	UP Diliman
15	Llemit, Dennis	M	Adamson University
16	Ona, Mark	M	UP Diliman
17	Oropeza, Alip	M	UP Diliman
18	Perez, Rolando III	M	UP Diliman
19	Saddi, Daryl Allen	M	UP Diliman
20	Santos, Jasmin-Mae	F	UP Diliman
21	Sato, Riuji	M	UP Diliman
22	Tirado, May Anne	F	UP Diliman
23	Vallejo, Louie John	M	UP Diliman
24	Velasco, Arrianne	F	UP Diliman