Olga Taussky, a Torchbearer for Mathematics and Teacher of Mathematicians

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Olga Taussky-Todd's mathematical and personal life (1906-1995), her achievements and obstacles, her scientific reasoning and teaching all have served as inspiration to many mathematicians. We describe her role in the mathematics world of the previous century as a torchbearer for mathematics and mathematicians, bearing the "torch of scientific truth" that burns inside of mathematics and its applications. Besides her many deep math contributions - too many to elaborate - she excelled at distilling and presenting mathematical concepts and ideas in her work and gave us many visionary papers and math talks. By sharing her mathematical vision freely she has inspired many of us.

1 Dates of Olga's life

August 30, 1906	Born in Olmütz, Austrian-Hungarian Empire, Mähren (Part of the Holy Roman (German) Empire prior to 1806; part of Czech Republic today); "Olga Taußky"
1909	Family moves to Vienna, father (chemist) dies early, 2 sisters
March 7, 1930	Ph.D. in Vienna under Phillip Furtwängler; on generalizations of Hilbert's Satz 94 (class field theory)
1931	Assistant in Göttingen (Courant)
1932	Co-editor (number theory portion) Hilbert's Collected Works
1934	Bryn Mawr College (with Emmy Noether) "Olga Taussky"
1935	Girton College, Cambridge
1937	University of London
1938	Marries Jack Todd (numerical analyst) (May 16, 1911 - June 21, 2007), "savior" of Oberwolfach in 1945
1947	National Bureau of Standards; (Institute of Advanced Studies)
1957	Research Associate at CalTech in Pasadena; from 1971: Professor
1958	Plenary hour lecture *"Integral Matrices"* at AMS; (second by a woman, first in 1934 by Emmy Noether)
1977	Professor Emeritus, CalTech; 19 Ph.D. students in class field theory, number theory, and matrix theory
October 7, 1995	Dies in Pasadena
2006	Most recent paper published (joint with J. Todd)

2 Areas of Olga's research

1. Algebraic number fields, class field theory, Hilbert Satz 94, Diophantine equations, Phytagorean numbers, sums of squares

2. Integral matrices, unimodular matrices, commutators, additive and multiplicative

3. Matrix eigenvalues, Gershgorin, matrix numerics, inertia theory, positive definite matrices, generalizations of Ljapunov

4. Matrices with property L or P, matrix pencils, similarity of A and A^* or A^T

186 entries in Zentralblatt (1930 - 2006); 175 entries in MR (1939 - 2006). [29,700 items in Google under "Olga Taussky"]

3 Olga in person

Olga finishes her Ph.D. at 23 years of age. She becomes one of the editors of Hilbert's Collected Works at age 26. She publishes almost 200 papers in 77 active years (1930 - 2006). She loves math throughout her life and works continually on math problems. She meets and interacts with everyone of the 20th century math community. She has a phenomenal memory of mathematics, of papers. authors, connections. She grows up in the German academic climate that at the time does not allow women's academic progress or female professors. She loves and works on math, numbers, finding mathematical relations, understanding the structure of math etc, every hour of her life. She is successful in her work, highly gifted, and widely admired. She helps others, is kind and supportive of the young. (Varga, Demmel ..) She finally becomes a Full Professor at CalTech in 1971 at age 65, when most professors retire. She is very patient with her situation of being female, Jewish and dedicates her efforts mainly to math research and to fostering mathematics and mathematicians.

Other than mathematics, one of Olga's personal joys was poetry. Here are two poems that deal with mathematics and mathematicians. The first one is patterned after a well known poem by Wilhelm Busch who invented comic strips in the 19th century and hails from her mathematical youth in Vienna and Göttingen.

Olga Taußky

Es steht die Olga vor der Klasse,
sie zittert sehr und denkt an Hasse;
die Emmy kommt von fern hinzu
mit lauter Stimm', die Augen gluh.

Die Trepp hinauf und immer höher
kommt sie dem armen Mädchen näher.
Die Olga denkt: weil das so ist,
und weil mich doch die Emmy frißt,

so werd' ich keine Zeit verlieren,
werd' keine Algebra studieren
und lustig rechnen wie zuvor.
Die Olga, dünkt mich, hat Humor.

There Olga waits before her classes,
she trembles much and thinks of Hasse's.
Miss Emmy comes from far aways,
her booming voice and glowing eyes.

Upstairs she climbs and still approaching,
she reaches poor young Olga crouching.
Thus Olga thinks: as this is so
and Emmy's sure to scold me oh,

I shan't continue wasting time
with algebraic studies mine,
but I'll return to fun and computations.
Miss Olga clearly laughs at her frustrations.

[German original from the 1930's; English translation by Frank Uhlig, 1996]

Wilhelm Busch, 1832 – 1908

Der Vogel auf dem Leim

Third poem in "Kritik des Herzens" (1874)

Es sitzt ein Vogel auf dem Leim,
Er flattert sehr und kann nicht heim.
Ein schwarzer Kater schleicht herzu,
Die Krallen scharf, die Augen gluh.

Am Baum hinauf und immer höher
Kommt er dem armen Vogel näher.
Der Vogel denkt: Weil das so ist,
Und weil mich doch der Kater frißt,

So will ich keine Zeit verlieren,
Will noch ein wenig quinquilieren
Und lustig pfeifen wie zuvor.
Der Vogel, scheint mir, hat Humor.

The next one shows her fully matured and enthusiastic for math.

Olga Taussky

Number Theory

Number theory seems greater
than what comes later
in the strict athletics
of mathematics.

For numbers can delight one
as was shown by Ramanujan
who could not prove all he found
and yet he knew it was sound.

Number theory is like poetry
they are both of the same kind
they start a fire in your mind.
Number theory is not just clever and smart
it has a beauty that fills your heart.

It is futile to wonder
whether far out and yonder
they have numbers that differ
from ours and obey rules
that seem strange and obscure
and yet have the same lure?

[from the 1960s]

4 Olga's growth; in mathematics and in person

Mathematically : Olga'a first, earliest, and foremost love in mathematics is number theory, algebraic number theory; and computing, or simply "lustig rechnen". Her number theory is algebraic, theoretic; but always based on the concrete. This desire for concreteness slowly turns her towards matrices. First to integer matrices, then to general matrices; and finally looking for structures of number theory in matrix theory and vice versa. She views matrix theory as concretized number theory and works to understand the two in a unified manner. She is a revolutionary soul in her mathematics that tries not to stir up personal controversy, but rather tries to explore and expose scientific truths.

First Math Example : *Gaussian elimination* is misnamed. Carl Friedrich Gauss came from geodesy and excessive data sets to the least squares problem. His contributions to solving linear systems lies in the "*normal equation*", not in inventing 'Gaussian' elimination. Gaussian elimination has been known since the ancient 9 books of Chinese mathematics, since Egypt, ... For hand computations in low dimensions, the "normal equation" approach is quite feasible for the least squares problem. In the late 1940s with the advent of early computers, larger overdetermined systems had to be solved. In England in 1949, Leslie Fox encountered previously unknown difficulties with computing least squares solutions via the normal equation. Matrix condition numbers had just been defined by von Neumann (1947) and Turing (1948).

What does Olga do here? She shows that Gauss's least squares normal equation method is unstable!

Theorem. [Math Tables, Aides to Computation, 4 (1950), 111-112]

Let A be a real n by n non-singular matrix and A' be its transpose. Then AA' is more "ill-conditioned" than A.

She gives proofs for both the P- and the N-condition number of a matrix and thus sets numerical linear algebra straight onto its modern road.

Second Math Example : The following both hold and are equivalent for an arbitrary field F:

(1) [Frobenius 1910, Taussky and Zassenhaus 1959]

Every n by n matrix A over F is similar via a symmetric matrix $S = S'$ over F to the transpose A' of A, i.e.,

$$A' \;=\; S^{-1}AS\;.$$

(2) [Taussky, Auburn 1970; (LAA 1972)]

Every n by n matrix A over F can be factored as $A = S \cdot T$ for $S = S'$, $T = T'$ over F and either one of the symmetric factors of A can be chosen nonsingular.

Proof: (1) $\Longrightarrow$ (2) : $S = S'$, $S^{-1}AS = A'$ imply $S^{-1}A = A'S^{-1} = (S^{-1}A)' = T$ is symmetric and $A = ST$.

(2) $\Longrightarrow$ (1) : $A = ST,\ S = S',\ T = T'$, and S nonsingular imply $S^{-1}A = T = T' = (S^{-1}A)' = A'S^{-1}$ or $S^{-1}AS = A'$.

Open Questions: How to make use of the symmetric factorization of square matrices :
(a) in applications, (b) in computations, (c) in numerical analysis, in algorithms, (d) in geometry, (e) in number theory.

Personally : Olga Taussky lives through difficult times, through two world wars and two emigrations. She stays true to her calling to be a mathematician by simply doing math. She attends every math colloquium that she can: in Vienna, Göttingen, in Princeton in the 1930s, in London in the 1940s, and in the LA area in the 1960s, 1970s, 1980s .. . But she always takes her notepad along to write down connections and ideas from the talk that she hears, or to work on her own math problems. She is quiet in public, she is not quick, and does not ask questions at talks. Everyone, however, asks her opinion in private on mathematics and on personal and professional matters. Throughout her life she adjusts and becomes a finely tuned creative *survivor* in many potentially dangerous environments. Therefore and thereby she holds onto math for all of her life and strives for scientific truths, regardless. For her love of mathematics and in order to fulfill her math dreams and visions, Olga foregoes all outward contentions and prefers to work studiously and quietly on her own art.

To be more than to appear "Mehr sein als scheinen"

The Todds, 1973